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The binding energy of the alpha particle with a velocity-dependent potential containing a tensor component

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Abstract. We make a variational calculation of the binding energy of the alpha particle with the velocity-dependent potential of Nestor *et al.* which contains a tensor component. The trial wave function used is a mixture of the $^1S_0$ and the principal $^5D_0$ states, the radial dependence of both these states being Gaussian. We calculate the Coulomb energy of interaction by the usual perturbation method with the potential of Schneider and Thaler which takes into consideration the finite size of the nucleons. Our calculation gives the values of the binding energy and the root-mean-square radius of the alpha particle in reasonably good agreement with experiments.

1. Introduction

During the past few years Green (1962), Razavy *et al.* (1957), Nestor *et al.* (1968) and others have shown that the hard core in the nucleon–nucleon potential can be replaced by a velocity-dependent potential. Such a velocity-dependent potential gives as good a fit to relevant two-body data as the hard-core potential does and has the additional advantage that perturbation methods can be used in the nuclear many-body problems.

Earlier Jain and Srivastava (1968) made a variational calculation of the binding energy of the alpha particle using a two-body central velocity-dependent potential. That calculation was not realistic in the sense that the actual two-body potential contains a strong tensor component. In the present investigation we make a variational calculation of the binding energy of the alpha particle using the velocity-dependent potential of Nestor *et al.* (1968) which contains a tensor component.

2. Calculation of the binding energy

The explicit form of the potential of Nestor *et al.* (1968) is given by

$$U_j = \frac{\hbar^2}{m}\left[V_j^{\mathrm{C}}(r) + \left\{\left(\frac{p^2}{\hbar^2}\right)\omega_j(r) + \omega_j(r)\left(\frac{p^2}{\hbar^2}\right)\right\} + V_j^{\mathrm{T}}(r)S_{12} + V_j^{\mathrm{LS}}(r)\boldsymbol{L}\,.\,\boldsymbol{S}\right] \tag{1}$$

in which the superscripts C, T and LS stand for central, tensor and spin–orbit forces respectively.

The velocity-dependent part is

$$\frac{\hbar^2}{m}\left\{\left(\frac{p^2}{\hbar^2}\right)\omega_j(r) + \omega_j(r)\left(\frac{p^2}{\hbar^2}\right)\right\} \tag{2}$$

and has been included only in the central part of the interaction. The tensor operator is

$$S_{12} = \{3(\boldsymbol{\sigma}_1\,.\,\boldsymbol{r})(\boldsymbol{\sigma}_2\,.\,\boldsymbol{r}) - r^2(\boldsymbol{\sigma}_1\,.\,\boldsymbol{\sigma}_2)\}/r^2. \tag{3}$$

The functions in equation (1) are

$$V_j(r) = -A_j \exp\left(-\frac{1}{2}\frac{r^2}{\alpha_j^2}\right) \tag{4}$$

$$\omega_j(r) = B_j \exp\left(-\frac{1}{2}\frac{r^2}{\beta_j^2}\right) \tag{5}$$

for all potentials except the tensor part $V_j^{\mathrm{T}}(r)$, for which

$$V_j^{\mathrm{T}}(r) = -A_j\left(\frac{1}{2}\frac{r^2}{\alpha_j^2}\right)\exp\left(-\frac{1}{2}\frac{r^2}{\alpha_j^2}\right). \tag{6}$$

The units of A_j are fm^{-2}, the B_j are dimensionless and $\hbar^2/m = 41{\cdot}47\ \mathrm{MeV\,fm}^2$. The index j stands for the four parts of the interaction: singlet–even, singlet–odd, triplet–even and triplet–odd.

We do not specify the potentials in the odd states since, in our calculation, we use a trial wave function which is symmetric in the spatial coordinates of all the four nucleons. In our calculation we use the set '*B*' of parameters in the potential of Nestor *et al.* (1968). The values of the potential parameters, in the singlet–even state, are

$$\left.\begin{aligned} &A_{\mathrm{C}}^{\mathrm{s}} = 2{\cdot}269\ \mathrm{fm}^{-2}, \quad \alpha_{\mathrm{C}}^{\mathrm{s}} = 0{\cdot}877\ \mathrm{fm}, \quad B^{\mathrm{s}} = 0{\cdot}6, \quad \beta^{\mathrm{s}} = 0{\cdot}877\ \mathrm{fm} \\ &A_{\mathrm{LS}}^{\mathrm{s}} = 0 \quad \text{and} \quad \alpha_{\mathrm{LS}}^{\mathrm{s}} = 0. \end{aligned}\right\} \tag{6a}$$

In the triplet–even state these are

$$\left.\begin{aligned} &A_{\mathrm{C}}^{\mathrm{t}} = 6{\cdot}825\ \mathrm{fm}^{-2}, \quad \alpha_{\mathrm{C}}^{\mathrm{t}} = 0{\cdot}598\ \mathrm{fm}, \quad B^{\mathrm{t}} = 1{\cdot}0, \quad \beta^{\mathrm{t}} = 0{\cdot}598\ \mathrm{fm} \\ &A_{\mathrm{T}} = 0{\cdot}490\ \mathrm{fm}^{-2}, \quad \alpha_{\mathrm{T}} = 1{\cdot}20\ \mathrm{fm}, \quad A_{\mathrm{LS}}^{\mathrm{t}} = 0 \quad \text{and} \quad \alpha_{\mathrm{LS}}^{\mathrm{t}} = 0. \end{aligned}\right\} \tag{6b}$$

The superscripts s and t stand for singlet and triplet states, respectively.

The trial wave function used is a mixture of the $^1S_0$ and the principal $^5D_0$ states (Irving 1953). The trial wave function may then be written as

$$\psi = \frac{1}{(1+C^2)^{1/2}}(\psi_{\mathrm{S}} + C\psi_{\mathrm{D}}). \tag{7}$$

In equation (7) ψ_{S} and ψ_{D} are separately normalized to unity, so that ψ is normalized to unity and C^2 determines the amount of D-state in the mixture. The complete wave functions ψ_{S} and ψ_{D} in the $^1S_0$ and the principal $^5D_0$ states, respectively, are the product of the radial wave functions and the corresponding angular and spin wave functions. Thus these wave functions are

$$\psi_{\mathrm{S}} = \phi_{\mathrm{S}}\chi \tag{8}$$

$$\psi_{\mathrm{D}} = \phi_{\mathrm{D}}\left\{\sum_{\substack{1\\ i<j}}^{4} \mathrm{D}(r_{ij})\right\}\chi = \phi_{\mathrm{D}}\left\{\sum_{\substack{1\\ i<j}}^{4} r_{ij}^2 S_{ij}\right\}\chi \tag{9}$$

where S_{ij} is the tensor operator and

$$\chi = \tfrac{1}{2}(\chi_1{}^{+}\chi_2{}^{-} - \chi_1{}^{-}\chi_2{}^{+})(\chi_3{}^{+}\chi_4{}^{-} - \chi_3{}^{-}\chi_4{}^{+}) \tag{10}$$

is the spin wave function. In equation (10) the subscripts 1, 2 denote the neutron and 3, 4 denote the proton coordinates.

We choose the form of the radial wave functions ϕ_S and ϕ_D to be Gaussian:

$$\phi_S = A_S \exp\left(-\tfrac{1}{2}\mu \sum_{\substack{1 \\ i<j}}^{4} r_{ij}^2\right), \qquad \phi_D = A_D \exp\left(-\tfrac{1}{2}\nu \sum_{\substack{1 \\ i<j}}^{4} r_{ij}^2\right). \tag{11}$$

Using the transformations (Irving 1953)

$$\boldsymbol{u} = \tfrac{1}{2}(\boldsymbol{r}_4+\boldsymbol{r}_3-\boldsymbol{r}_2-\boldsymbol{r}_1), \qquad \boldsymbol{v} = (\boldsymbol{r}_2-\boldsymbol{r}_1)/\sqrt{2}, \qquad \boldsymbol{w} = (\boldsymbol{r}_4-\boldsymbol{r}_3)/\sqrt{2}$$

and

$$\boldsymbol{R} = \tfrac{1}{4}(\boldsymbol{r}_1+\boldsymbol{r}_2+\boldsymbol{r}_3+\boldsymbol{r}_4) \tag{12}$$

we get

$$\psi_S = N_S \exp\{-2\mu(u^2+v^2+w^2)\}\chi \tag{13}$$

and

$$\psi_D = N_D \exp\{-2\nu(u^2+v^2+w^2)\}\{6(-\boldsymbol{\sigma}_1 \cdot \boldsymbol{v})(\boldsymbol{\sigma}_3 \cdot \boldsymbol{w})+6(\boldsymbol{\sigma}_1 \cdot \boldsymbol{w})(\boldsymbol{\sigma}_3 \cdot \boldsymbol{v}) - 4(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_3)(\boldsymbol{v} \cdot \boldsymbol{w})\}\chi. \tag{14}$$

The normalization constants in the new coordinate system are found to be given by

$$N_S = 2^{9/2}\left(\frac{\mu}{\pi}\right)^{9/4}, \qquad N_D = \left(\frac{2^{11}}{45}\right)^{1/2}\left(\frac{\nu^{13}}{\pi^9}\right)^{1/4}. \tag{15}$$

We calculate the energy of the alpha particle, excluding the Coulomb energy which is treated as a perturbation, using the Rayleigh–Ritz variational principle, by minimizing the expectation value of the Hamiltonian, $\langle H\rangle = \int \psi^* H \psi \, d\tau$, with respect to the variational parameters, μ, ν and C. Now, apart from the Coulomb energy,

$$\langle H\rangle = W_{^4\mathrm{He}} = \langle T\rangle_{^4\mathrm{He}} + \langle V^{\mathrm{C}}_{\mathrm{static}}\rangle_{^4\mathrm{He}} + \langle V^{\mathrm{C}}_{\mathrm{vel\ dep}}\rangle_{^4\mathrm{He}} + \langle V^{\mathrm{T}}_{\mathrm{static}}\rangle_{^4\mathrm{He}}. \tag{16}$$

We follow Irving (1953) in evaluating the various energy matrix elements in equation (16).

The Coulomb energy of interaction of the protons in the alpha particle is calculated by the perturbation method using the potential

$$V_{pp} = \frac{e^2}{r}[1-\{\exp(-3{\cdot}36r)\}(0{\cdot}528r-2{\cdot}776)-\{\exp(-2{\cdot}97r)\}(0{\cdot}644r+3{\cdot}639)] \tag{17}$$

of Schneider and Thaler (1965), which takes into consideration the finite size of nucleons. This potential gives for the Coulomb energy of the protons in the alpha particle

$$E_{\mathrm{Coul}} = \int \psi^* V_{pp}(r_{34})\psi \, d\tau. \tag{18}$$

3. Root-mean-square charge radius of the alpha particle

The mean-square charge radius of the alpha particle considering point protons is given by

$$\langle r^2\rangle_{00} = \frac{1}{Z}\left\langle \sum_{p} (\boldsymbol{r}_p-\boldsymbol{R})^2 \right\rangle_{00}. \tag{19}$$

In the above equation, $\boldsymbol{r}_p$ denotes the proton coordinates and $\boldsymbol{R}$ denotes the centre-of-mass coordinates.

Using equation (12), we get

$$\langle r^2 \rangle_{00} = \langle \tfrac{1}{4}u^2 + \tfrac{1}{2}w^2 \rangle_{00} \tag{20}$$

and a simple integration gives the mean-square charge radius

$$\langle r^2 \rangle_{00} = \frac{1}{1+C^2}\left(\frac{9}{32\mu} + \frac{13C^2}{32\nu}\right). \tag{21}$$

Using the best values of parameters, μ, ν and C, obtained from the variational calculation of the binding energy, we find

$$\langle r^2 \rangle_{00} = 1{\cdot}66 \text{ fm}^2. \tag{22}$$

The mean-square charge radius $\langle r^2 \rangle_{00}$ is related to the mean-square charge radius R_c^2 of the alpha particle measured from electron scattering experiments through the equation

$$\langle r^2 \rangle_{00} = R_c^2 - R_p^2 \tag{23}$$

where R_p is the root-mean-square radius of the charge distribution of the proton.

We use equation (23) to calculate $(\langle r^2 \rangle_{00})_{\text{expt}}$ from the values of $R_c = 1{\cdot}61$ fm and $R_p = 0{\cdot}8$ fm obtained from electron scattering experiments and to compare with our calculated value as given by equation (22). The results are shown in the column 3 of table 1.

Table 1. Binding energy and r.m.s. charge radius of the alpha particle

Reference	Binding energy (MeV)	Coulomb energy (MeV)	r.m.s. charge radius (fm)	Best values of the parameters in the trial wave function		
				μ(fm^{-2})	ν(fm^{-2})	C
Present calculation	22·88	0·824	1·29	0·17	0·25	−0·11
Jain and Srivastava[1]	30·12	0·826	1·55			
Herndon *et al.*[2]	31·0		1·31			
Irving[3]	31·9	1·0	1·29			
Irving[4]	24·2	1·30	0·99			
Tang *et al.*[5]	30·97					
Experimental	28·3		1·45 ±0·03			

[1] Using a central velocity-dependent potential (Jain and Srivastava 1968).
[2] Using a central velocity-dependent potential (Herndon *et al.* 1963).
[3] Using static central potential without a core (Irving 1951).
[4] Using a static tensor potential without a core (Irving 1953).
[5] Using a central hard-core potential (Tang *et al.* 1965).

4. Discussion

Our variational calculation gives a value of 22·88 MeV for the binding energy of the alpha particle which is lower than the experimental value (28·3 MeV). This is in agreement with the variational principle according to which the calculated value of the binding energy W is less than or equal to E, the experimental value, provided the correct interaction is used. In our case, it is possible to reduce the discrepancy between the calculated and experimental values by choosing better trial wave functions. A variational calculation with a trial wave function which takes into account correlations of nucleons is in progress and will be reported later.

A comparison between the results of earlier calculations (without hard core or velocity dependence) and the experiment shows that the alpha particle with such potentials is overbound (Irving 1951). Introduction of hard core or velocity-dependent forces causes the binding energy to change in the right direction but the alpha particle remains still slightly overbound (Jain and Srivastava 1968, Herndon *et al.* 1963 and Tang *et al.* 1965). Our calculation shows that tensor forces, as expected (Blatt and Weisskopf 1952) further reduce the binding energy by introducing D-states and the binding energy becomes consistent with the variational principle. This clearly, though indirectly, indicates that two-body nuclear forces must have a tensor component.

Using the perturbation method of Bolsterli and Feenberg (1956), Goldhammer and Valk (1962) have calculated the binding energy of the alpha particle with a repulsive core potential containing a tensor component. Though they obtain a very good value for the binding energy (28·4 MeV), their value for the D-state probability is unusually large (10·6%).

In connection with earlier calculations of the binding energy of the alpha particle it has been noted by Rustgi and Levinger (1957) and independently by Dalitz and Ravenhall (Hofstadter 1956) that though the agreement with the binding energy was good, the root-mean-square radius was only 2/3 of the experimental value. It is satisfactory to note that we get reasonably good values both for the binding energy and the root-mean-square radius.

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